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$$\begin{aligned}
 \alpha &= 180^\circ - A = 110^\circ \\
 \beta &= 180^\circ - B = 48^\circ 50' \\
 \gamma &= 180^\circ - C = 85^\circ 10' \\
 2\sigma &= 244^\circ \\
 \sigma &= 122^\circ \\
 \sigma - \alpha &= 12^\circ \\
 \sigma - \beta &= 73^\circ 10' \\
 \sigma - \gamma &= 36^\circ 50'
 \end{aligned}$$

To find $\log \tan \frac{1}{2} \delta$

$$\begin{aligned}
 \log \sin(\sigma - \alpha) &= 9.3179 \\
 \log \sin(\sigma - \beta) &= 9.9810 \\
 \log \sin(\sigma - \gamma) &= 9.7778 \\
 \text{colog sin } \sigma &= 0.0716 \\
 \hline
 & 2|9.1483 \\
 \log \tan \frac{1}{2} \delta &= 9.5742
 \end{aligned}$$

$$\begin{aligned}
 \log \sin(\sigma - \alpha) &= 9.3179 \\
 \log \tan \frac{1}{2} \delta &= 9.5742 \\
 \log \tan \frac{1}{2} a &= 9.7437 \\
 \frac{1}{2} a &= 39^\circ \\
 a &= 58^\circ
 \end{aligned}$$

$$\begin{aligned}
 \log \sin(\sigma - \beta) &= 9.9810 \\
 \log \tan \frac{1}{2} \delta &= 9.5742 \\
 \log \tan \frac{1}{2} b &= 0.4068 \\
 \frac{1}{2} b &= 68^\circ 36' \\
 b &= 137^\circ 12'
 \end{aligned}$$

$$\begin{aligned}
 \log \sin(\sigma - \gamma) &= 9.7778 \\
 \log \tan \frac{1}{2} \delta &= 9.5742 \\
 \log \tan \frac{1}{2} c &= 0.2036 \\
 \frac{1}{2} c &= 57^\circ 58' \\
 c &= 115^\circ 56'
 \end{aligned}$$

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

308. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Find the conditions that the roots of $x^2 + px + q = 0$ may not lie between -1 and $+1$.

I. Solution by J. A. CAPARO, University of Notre Dame, Notre Dame, Ind.

In the most general case let one of the roots be $+a$ and the other $-b$, then:

$$(x-a)(x+b)=0 \text{ or } x^2 + x(b-a) - ab = 0.$$

Comparing with $x^2 + px + q = 0$, $p = b - a$, $q = -ab$.

The conditions that the roots shall not lie between $+1$ and -1 are:

$$+a > 1 \dots (1); \quad +b > 1 \dots (2).$$

Multiplying, $ab > 1$, but $ab = -q$, therefore $-q > 1$ or $q < -1$. Also from (1), (2), $a-1 > 0$, $b+1 > 2$.

Multiplying, $ab - b + a - 1 > 0$, or $-ab + (b-a) + 1 < 0$, or $+q + p < -1$. The required conditions then are: $q < -1$ and $q + p < -1$.

II. Solution by S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

Assuming that the equation has real roots, *i. e.*, that $p^2 > 4q$, find Sturm's functions. They are,

$$\left. \begin{array}{l} f(x) = x^2 + px + q \\ f'(x) = 2x + p \\ f_2(x) = p^2 - 4q \end{array} \right\}$$

For +1, we have, $f(x) = 1 + p + q$, $f_1(x) = 2 + p$, $f_2(x) = p^2 - 4q$.

For -1, we have, $f(x) = 1 - p + q$, $f_1(x) = -2 + p$, $f_2(x) = p^2 - 4q$.

If there is no root between +1 and -1, there will be the same number of variations of signs in each of these. $p^2 - 4q$ is always positive. We see that the number of variations will be the same if $f(1)$ and $f(-1)$, $f'(1)$ and $f'(-1)$ have the same sign, *i. e.*, $(1+q)^2 - p^2 > 0$ and $p^2 - 4 > 0$, *i. e.*,

$$(1+q)^2 > p^2 > 4.$$

They will also have the same number of variations if $f'(1)$ and $f'(-1)$ have opposite signs and $f(1)$ and $f(-1)$ are both negative, *i. e.*, $p^2 < 4$, $1+q \pm p < 0$, or $p \pm q > 1$.

Also solved by G. B. M. Zerr, V. M. Spunar, and G. W. Hartwell.

309. Proposed by PROFESSOR E. B. ESCOTT, Ann Arbor, Mich.

Solve, $bx^2 + cy^2 + az^2 = ba^2 + cb^2 + ac^2$,
 $cx^2 + ay^2 + bz^2 = ab^2 + bc^2 + ca^2$,
 $xyz = abc$.

Solution by the PROPOSER.

We see by inspection that $x=a$, $y=b$, $z=c$ is one set of solutions.

Then, put $x^2 = a^2 + u$, $y^2 = b^2 + v$, $z^2 = c^2 + w$. The first two equations become $bu + cv + aw = 0$, $cu + av + bw = 0$. Solving these, we have

$$\frac{u}{a^2 - bc} = \frac{v}{b^2 - ca} = \frac{w}{c^2 - ab}.$$

Put these equal to s , and substitute

$$\begin{aligned} u &= (a^2 - bc)s, \\ v &= (b^2 - ca)s, \\ w &= (c^2 - ab)s, \end{aligned}$$

in values of x^2 , y^2 , and z^2 :

$$\begin{aligned} x^2 &= a^2 + (a^2 - bc)s, \\ y^2 &= b^2 + (b^2 - ca)s, \\ z^2 &= c^2 + (c^2 - ab)s. \end{aligned}$$